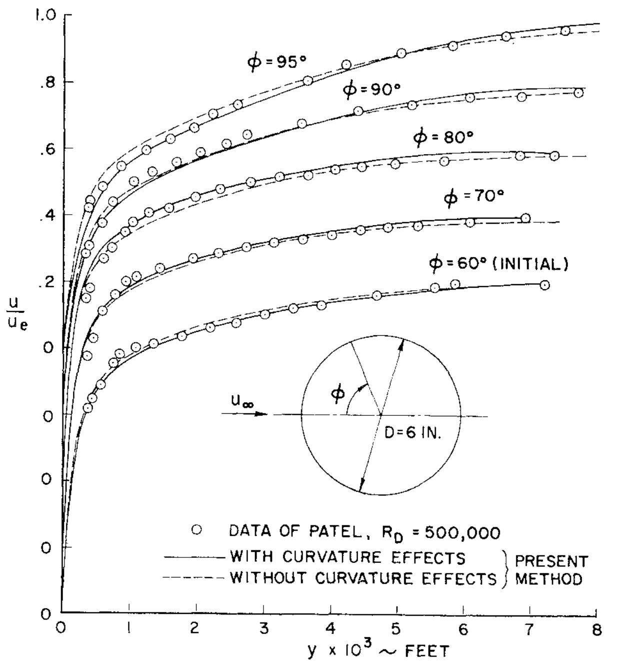

Fig. 3 Comparison of calculated velocity profiles and data of Patel.[12]

The present turbulent flow results have made use of Richardson-number modifications to the eddy viscosity with a value of the curvature parameter, which is of similar order of magnitude to that used in previous investigations.[8] The approximate nature of the equations and physical assumptions insures that there can be no unique value of this parameter which will allow all flows to be represented exactly. With the present equations and the two comparisons made, the value of 3.5 appears to be best, but a single value of this order is likely to produce accurate results over a wide range of flows. The use of higher-order turbulence models is difficult to justify for this purpose, as suggested by Bradshaw.[13] Additional constants are required to represent the curvature in additional conservation equations, which themselves involve significant and untested approximations.

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Subsonic Base Pressure Fluctuations

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Nomenclature

C_{P_b} = base pressure coefficient $C_{P_b} = 2(P_b - P_1)/\rho U^2$
D = model diameter
f = frequency
M_1 = freestream approach Mach number
P_1 = freestream pressure
P_b = base pressure
P_b' = fluctuating component of base pressure
P_{atm} = atmospheric pressure
U = freestream approach velocity
S = Strouhal number, $S = fD/U$
ρ = density

Introduction

THE abrupt change in the rear geometry of a bluff body moving through a real fluid causes the external flow to separate from the body. This separated region, which occurs at or near the base of the body, is usually referred to as the near-wake of the body. The near-wake is dominated by the mixing process associated with the free shear layer which results from the flow separation. While not large, this zone has a significant influence on base drag, base heat transfer, and the configuration of the far-wake. The components which comprise the near-wake flowfield of a blunt axisymmetric body are shown in Fig. 1.

The subsonic base flow problem has been the subject of a great deal of study in recent years. Over the past thirty years, there have been many experimental investigations of the wakes of various axisymmetric bodies and a number of theoretical analyses of the problem. Rather complete reviews of the available literature may be found in Refs. 1 and 2.

One of the most common near-wake parameters reported in the literature is base pressure. In general, the base pressure is presented as a steady-state or time-averaged quantity and little or no mention is made of the base pressure oscillations. However, a knowledge of the base pressure fluctuation is as important as a knowledge of the time-averaged base pressure. The fluctuating component of the base pressure is of prime concern in the consideration of buffet and vibration of blunt-based bodies of revolution. Some data on base pressure fluctuation on various bodies of revolution have been presented by Eldred[3] and Mabey.[4] This Note presents some

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Aeroacoustics; Viscous Nonboundary-Layer Flows.

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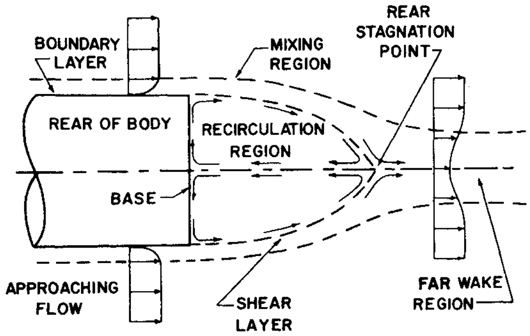

Fig. 1 Flowfield schematic.

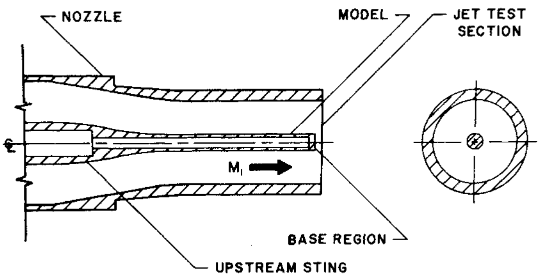

Fig. 2 RANT II schematic.

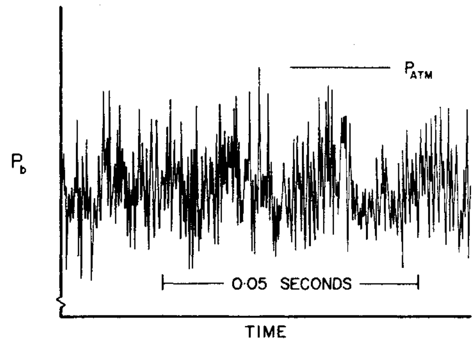

Fig. 3 Typical base pressure fluctuations.

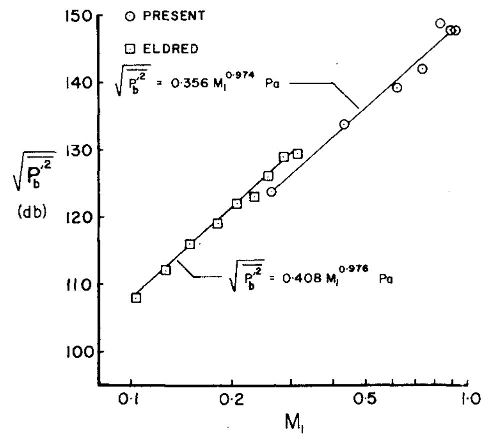

Fig. 4 Magnitude of base pressure fluctuations vs Mach number.

additional preliminary information concerning base pressure oscillations on an axisymmetric blunt-based right-circular cylinder aligned with the flow direction (Fig. 1). The data presented have been obtained over the entire subsonic speed range in a special facility which eliminates model support interference.

Experimental Apparatus and Technique

A series of tests were conducted in a second-generation Rutgers Axisymmetric Near-Wake Tunnel (RANT II). This tunnel is an open jet, blowdown-type facility capable of producing speeds over the entire subsonic Mach number range. RANT II was designed and constructed for interference-free studies of turbulent axisymmetric near-wakes at subsonic speeds. As shown in Fig. 2, the unique feature of RANT II is an upstream sting which was designed as an integral part of the nozzle to produce uniform flow over a 1.9 cm diam cylindrical model. The nozzle has an overall contraction ratio of 8:1 and an exit diameter of 10.16 cm. A complete description of the facility may be found in Ref. 1.

Preliminary measurements were made to insure that the tunnel produced a satisfactory flowfield for the present investigation. These measurements showed that: 1) the freestream was uniform and axisymmetric to within 0.5% of the freestream velocity at all Mach numbers; 2) at all speeds, the Mach number was constant in the axial direction to within 1% for a distance of ±4 model diameters of the nozzle exit; and 3) the core region of the free jet extended over twenty model diameters downstream of the blunt base and was of sufficient size so that meaningful near-wake data could be obtained.

The fluctuations in base pressure were measured using a Kulite semiconductor pressure transducer, model XT-190-5, which was flush-mounted on the base of the model. This transducer has a frequency response of 70,000 Hz and a pressure range of ±34.5 kPa gage. The signal from the transducer was amplified and recorded on a light beam oscillograph which had a frequency response of 15,000 Hz.

Test conditions for this investigation included: 1) the entire subsonic Mach number range, 2) Reynolds numbers from 0.1×10^7 to 3.1×10^7 per meter, 3) stagnation pressures from just above atmospheric pressure to 206 kPa absolute, and 4) stagnation temperatures of 283 K ± 11 K. Velocity profiles of the boundary layer approaching the blunt base showed that it was turbulent and fully developed for all approach Mach numbers. Hot-wire probe measurements indicated that the turbulence level of the freestream was 0.6%. The turbulence level in the boundary layer approaching the blunt base was approximately 8%.

Results

A trace of the base pressure with time is shown in Fig. 3. This trace is typical of the base pressure fluctuations observed at all the Mach numbers tested. It is quite obvious that the base pressure is not constant with time, but rather exhibits large and rapid fluctuations about a mean value.

The time-averaged characteristics for the subsonic axisymmetric base flow problem have been reported.[5] For Mach numbers between 0.0 and 0.8, the time-averaged base pressure coefficient C_{P_b} was nearly constant and equal to -0.11. A very slight increase in C_{P_b} was observed between the Mach numbers of 0.4 and 0.6. At Mach numbers above 0.8, the base pressure coefficient began to fall rapidly as near-sonic speeds were approached.

The fluctuating component of the base pressure was time-averaged and an rms value for the magnitude of the fluctuations was obtained at each Mach number. These results are shown in Fig. 4 along with the results of Eldred.[3] When plotted in log-log form (Fig. 4), the data exhibit a relatively linear relationship. A straight line was fit to each of the sets of data by a least-squares method. As shown in Fig. 4, these two lines have nearly the same slope and differ only by a constant. Thus, the functional relationship with Mach number is the same for both sets of data and only the magnitudes of the fluctuations differ for the two cases at a given Mach number. This magnitude difference is probably attributable to differences in the flow conditions for the two sets of data. In particular, the overall total pressure level of each flow, as well as the turbulence intensity of the approaching freestream and

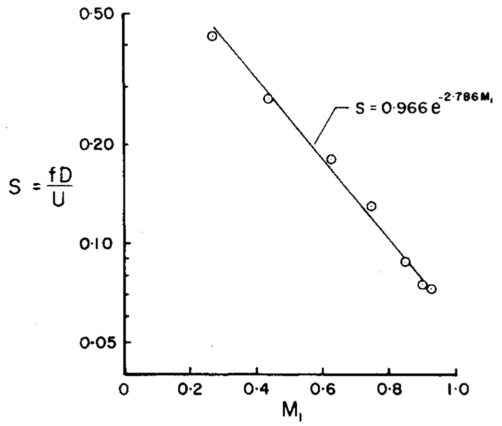

Fig. 5 Strouhal number vs Mach number.

boundary layer, may account for the difference in fluctuation magnitude for the two cases.

Although the equipment necessary to obtain detailed frequency information concerning the base pressure fluctuation was not available, some results can be drawn from the data. A simple count of the number of fluctuations in base pressure that occurred during an extended period of time yielded an average value for the frequency of the fluctuations. This average value of frequency gives a rough estimate of the frequencies at which that portion of the signal with significant amplitudes occur. In general, the average frequency decreased with increasing Mach number from approximately 2000 Hz at the low Mach numbers to approximately 1000 Hz at near-sonic speeds. A Strouhal number based on the average frequency, the model diameter, and the freestream velocity was computed. The Strouhal number as a function of Mach number is shown in Fig. 5. A linear relationship is evident on the semilog plot and a straight line, which was fit to the data by a least-squares method, is shown in the figure.

Conclusions

The base pressure fluctuations on an axisymmetric blunt-based body at subsonic speeds have been investigated. The tests were conducted over the entire subsonic Mach number range in a special wind tunnel that was free of support interference. The magnitudes of the fluctuations were found to be significant and increased with increasing Mach number. Magnitudes between 120 dB and 150 dB were observed. An average frequency and Strouhal number were found to decrease with increasing Mach number. Average frequencies between 1000 Hz and 2000 Hz were observed, while Strouhal numbers were between 0.05 and 0.5.

Acknowledgement

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Some Exact Solutions to Guderley's Equation

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Introduction

THE transonic small disturbance equation $(K-(\gamma+1)\varphi_x)\varphi_{xx}+\varphi_{yy}(x,y)=0$, in the usual notation, can be transformed into $-(\gamma+1)\phi_x\phi_{xx}+\phi_{yy}(x,y)=0$ using $\varphi=Kx/(\gamma+1)+\phi$. This latter equation is considered at great length in Guderley's classic monograph,[1] where various similar, separable and hodograph solutions are presented. In this Note, we re-examine Guderley's solution $\phi=x^3f(y)$ for the parallel sonic jet, and consider the broader class of separable solutions $\phi=g(x)f(y)$. Despite the apparent simplicity, solutions taking this form have not, to the author's knowledge, been treated previously. Guderley assumes $\phi=x^nf(y)$ and determines the exponent n so that separability is insured. The broader class of jetlike solutions considered here describes flows with more general expansion rates and should be of special engineering interest. In addition, the solutions provide useful test cases for numerical methods.

Analysis

The foregoing Ansatz leads to two ordinary nonlinear differential equations; namely,

$$f''-(\gamma+1)\lambda f^2=0 \tag{1}$$

$$g'g''-\lambda g=0 \tag{2}$$

where λ is a separation constant. Several solutions are possible. For example, two functions satisfying Eq. (1) are defined from:

$$f_1(y)=6/[\lambda(\gamma+1)]y^{-2} \tag{3}$$

and

$$\pm\int_{f^*}^{f_2}\frac{d\xi}{\sqrt{2\alpha+2/3\lambda(\gamma+1)\xi^3}}=y-y^* \tag{4}$$

where, in Eq. (4), f^* is the value of $f_2(y)$ at $y=y^*$ and α is a constant. Equation (2), on the other hand, is solved by $g_1(x)$ and $g_2(x)$, as determined from:

$$g_1(x)=(\lambda/18)x^3 \tag{5}$$

and

$$\int_{g^*}^{g_2}\frac{d\xi}{(3\beta+3/2\lambda\xi^2)^{1/3}}=x-x^* \tag{6}$$

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